Measuring the Circumference of the Earth with GPS

In this activity, learners measure the circumference, radius, mass, and volume of the Earth using Global Positioning Systems (GPS) receivers. This activity incorporates and integrates geography, mathematics, Earth Science, and physics. It can be used with secondary learners through university level and adult learners. The activity requires approximately 90 minutes to complete at the minimum and 150 minutes at the maximum. The field work can be done in as little as 20 minutes and because it only requires a small area, can be done on the school campus grounds.

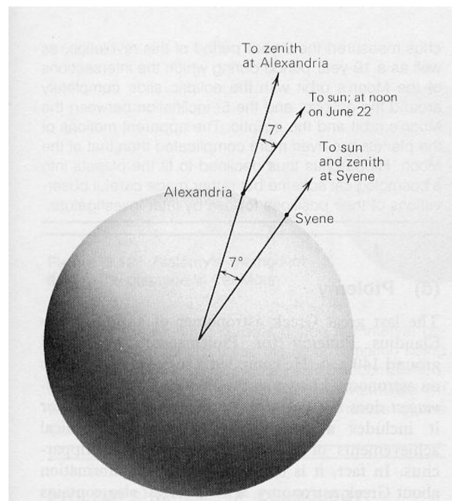

Figure 2.9 Eratosthenes' method of determining the size of the Earth.

Eratosthenes (276 BC-194 BC) was born in Cyrene, now a part of Libya, in the northern part of Africa. After studying in Alexandria and Athens, he became the director of the Great Library in Alexandria. The Library truly lived up to its name, housing a great deal of the learned and compiled knowledge of the time. It was at the library where Eratosthenes read about a deep vertical well near Syene (now Aswan) in southern Egypt. Once a year at noon at this well, on the day of the Summer Solstice, the bottom of the well was entirely lit up by the sun. The sun was directly overhead, its rays shining straight into the well.

Eratosthenes then placed a vertical post at Alexandria, which was almost due north of Syene, and measured the angle of its shadow on the same date and time. Making the assumptions that (a) the earth is round and that (b) the sun's rays are essentially parallel, Eratosthenes knew from geometry that the size of the measured angle equaled the size of the

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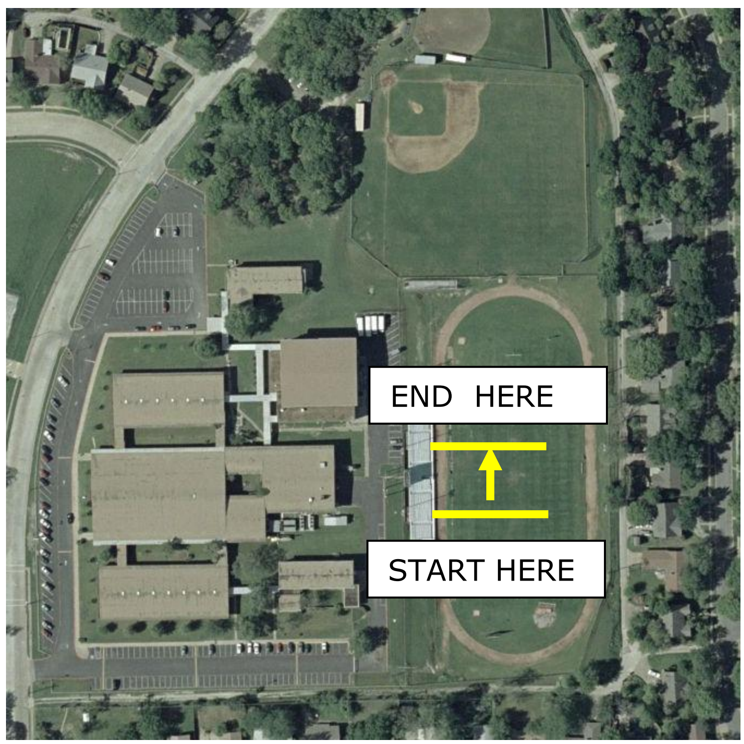
angle at the earth's center between Syene and Alexandria. Knowing also that the arc of an angle this size was 1/50 of a circle, he then had to determine the distance between Syene and Alexandria. This was a difficult task during that time, due to different strides of camels and human error, and despite the best efforts of the King's surveyors, required years of effort. It was finally determined to be 5000 stadia. Eratosthenes multiplied 5000 by 50 to find the Earth's circumference. His result, 250,000 stadia (about 46,250 km), was amazingly close to the accepted modern measurements (40,075 km around the equator and 40,008 km around the poles).

With your GPS, **you too** can emulate Eratosthenes' methods and measure the circumference of the Earth! By so doing, you are incorporating and integrating geography, mathematics, Earth Science, and physics!

Measuring the Earth's Circumference

1. Gather in pairs. Each person needs to have a GPS receiver. Alternatively, you can do this with a smartphone app that can give its position in decimal degree AND in UTM coordinates. Ideally, each person has their own smartphone.



2. For each pair of learners, set one GPS receiver to display coordinates in the format of degrees minutes seconds (hddd° mm' ss.s") latitude-longitude coordinates. The "H" stands for Hemisphere, ddd=degrees, learnermm=minutes, and ss=seconds. Set the other GPS receiver to UTM (Universal Transverse Mercator). The UTM units are in meters. The units in the UTM system represent eastings ("east" relative to a Central

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Meridian in given UTM zone), and northings (meters north of the Equator). Important: Set each GPS receiver's datum to WGS 84 so that all learners are working with the same datum (model of the Earth's shape). Otherwise, coordinates for the two learners in each pair could be vastly different.

3. Have the learners use the GPS compass to determine which direction true north and true south lie from their present position.

4. Position the learners until they are located at a whole second of latitude (not a fraction), as measured by the learner using the degrees-minutes-second format. For example: 39 degrees, 20 minutes, and 5 seconds, rather than 39 degrees, 20 minutes, and 5.1 seconds. At this whole-second position, write down the coordinates that are showing on each GPS receiver (or mark a waypoint). This will be latitude-longitude degrees-minutes-seconds on one GPS and UTM on the other GPS.

5. Along the north-south line, have each pair of learners walk due north or due south for 1 full second of latitude. When a full second of latitude has been traversed, the learner with the latitude-longitude GPS needs to call "STOP".

6. When the learners stop, write down the coordinates showing on each GPS receiver (or mark a waypoint). Again, this will be lat-long degrees-minutes-seconds on one GPS and UTM on the other GPS.

7. Determine how many meters the pair has walked by comparing the UTM northing at the starting point to the UTM northing at the end point:

_______________ meters

8. Compute the Earth's circumference by using the following equations:

1 second of latitude x 60 = 1 minute of latitude
1 minute of latitude x 60 = 1 degree of latitude

1 degree of latitude x 360 = the number of degrees around the Earth

Therefore, the number of meters that the learners have recorded above x 60 = the number of meters in 1 minute.

The number of meters in 1 minute x 60 = the number of meters in 1 degree.

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The number of meters in 1 degree x 360 = the number of meters around the Earth, through the poles.

Fill in the following:

The number of meters that you walked = __________

x 60 = __________ meters in 1 minute

x 60 = __________ meters in 1 degree

x 360 = __________ meters around the Earth

Divided by 1000 = __________ kilometers around the Earth

9. How close are you in percent to the accepted circumference of the Earth? Show your work.

10. Are you closer to the accepted circumference than Eratosthenes was?

11. Name at least 3 reasons why your answer is not exactly the same as the accepted circumference of the Earth.

Example:

The number of meters that you walked =

4391181 – 4391150 = 31 meters

x 60 = 1,860 meters in 1 minute

x 60 = 111,600 meters in 1 degree

x 360 = 40,176,000 meters around the Earth

Divided by 1000 = 40,176 kilometers around the Earth

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Error: 40,176 – 40,008 = 168 / 40,008 = .004 x 100 = .4 %.

How Long Would It Take to Walk Around the Earth?

1. Set the GPS receiver to the screen where you can determine how fast you are moving.

2. How fast are you walking (in km/hour) when you are walking at a comfortable pace?

3. If you could keep up this pace and walk due north or due south from this point, and walk all the way around the Earth on the meridian, how long would it take before you arrived back at this same spot? Show your work.

4. What would the date be when you arrived back at this spot? Show your work.

Example:

$$= \frac{40{,}176 \text{ km}}{6 \text{ km / hour}}$$

= 6,696 hours, or 279 days.

Determining the Earth's Circumference Based on Measuring 1 Second of Longitude

1. Pair up the learners as before, but this time, position them at a whole second of *longitude*.

2. Record the latitude that you are standing on. Convert this latitude value to decimal degrees: Latitude = degrees + minutes/60 + seconds/3600

3. Walk due east or west for a distance of exactly one second of longitude.

4. Record the distance walked.

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5. Use the following formula to compute the Earth's polar circumference. When doing the calculation, make sure the cosine is measured in degrees, not radians (otherwise, your final value will be a negative number!).

$$\frac{\text{Distance walked in meters} * 360}{\text{Distance walked in degrees}} * \frac{1}{\text{cosine of latitude } \varnothing}$$

Example:

$$\frac{24 * 360}{.0002777} * \frac{1}{\cos(40)} = \frac{8640}{.0002777} * \frac{1}{.766} = 40{,}617{,}116 \text{ meters}$$

Determining the Distance Around the Earth At Your Present Latitude

Method A

1. Determine the length of 1 second of longitude using the method above.

2. Compute:

$$\frac{\text{Distance walked in meters} * 360 * (\text{cosine of latitude})}{\text{Distance walked in degrees}}$$

Example:

$$\frac{24 \text{ m} * 360 * (\text{cosine of latitude})}{.0002777}$$

= 31112711 * .766

= 23,832,336 meters, or 23,832 kilometers

3. Why is the distance around the Earth on this latitude line shorter than the distance around the Earth along this longitude line?

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4. On what line on the Earth is the distance around the Earth on a latitude line close to the distance around the Earth on a longitude line?

Method B

1. Determine the length of the polar circumference using the method above.

2. Compute: length of this latitude = cosine of latitude * length of Equatorial circumference

3. Is the length of this latitude longer or shorter than the equatorial circumference? Why?

4. Determine the length of time it would take for you to walk around the Earth at this latitude at a comfortable walking speed.

5. What would the date be when you arrived back at this spot? Show your work.

6. Is the time it takes to walk around the Earth at this latitude east or west longer or shorter than it would take to walk around the Earth due north or south? Why?

Determining the Mass of the Earth

1. Determine the length of the polar circumference using the method above.

2. Determine the Earth's radius based on the circumference.

3. Compute:

Mass = acceleration * radius2/ G

where acceleration due to gravity = 9.8 meters2, and G, the constant of proportionality, which was computed by Henry Cavendish in 1798, is 6.67 x 10-11 / kg seconds2.

For example: Radius = 6.4 x 106 meters

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M = 9.8 * (6.4 * 106) / 6.7 x 10-11 kg

M = 6.0 x 1024 kg

Determining the Volume of the Earth

1. Determine the length of the polar circumference using the method above.

2. Determine the Earth's radius based on the circumference.

3. Compute the Earth's volume from the formula below:

Volume = 4/3 * π * r3

For More Information

Read the book "The Librarian Who Measured the Earth" by Kathryn Lasky. Published by Little, Brown, and Company:
http://www.amazon.com/Librarian-Who-Measured-Earth/dp/0316515264

Excellent for all ages but geared for learners aged 7-12.
Read more about Eratosthenes on:
http://www.eranet.gr/eratosthenes/html/eoc.html

and
http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Eratosthenes.html

Read the book *The Mapmaker's Wife* by Robert Whitaker, an amazing tale about measuring 1 degree of latitude in Peru in 1730 by a French mapmaking team.

Read the book *Longitude* by Dava Sobel, a fascinating book about the man who invented the world's first accurate clock so that ship captains could determine their position at sea. A tale about the triumph of the common man!

Read the book *The Mapmakers* by John Noble Wilford, a book about the history of cartography over the centuries.

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